Quadcopter stabilization by using PID controllers

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Fecha de recepción: 21 de septiembre de 2014 - Fecha de aceptación: 17 de octubre de 2014

RESUMEN

En este trabajo se presenta la estabilización de un cuadricóptero mediante el uso de controladores PID para la regulación de sus 4 movimientos básicos: ángulos de alabeo, cabeceo, guiñada y altitud. La estabilización del sistema se realiza a partir de un modelo simplificado que facilita la implementación de controladores en un sistema SISO, demostrando efectividad dentro de un rango de ±10° para los ángulos de alabeo y cabeceo y rango completo en guiñada y altitud. El módulo del controlador PID es diseñado para ser usado en cuadricópteros comerciales y ha sido implementado en base a sensores inerciales y ultrasónicos. Además, el sistema cuenta con una interface para observar el desempeño de la aeronave durante el vuelo.

Palabras clave: Cuadricóptero, sistemas de control, controladores PID, UAV.

ABSTRACT

This work presents the stabilization of a quadcopter by using PID controllers to regulate its four basic movements: roll, pitch, yaw angles, and altitude. The stabilization of the system is made from a simplified model which makes easier the implementation of controllers on a SISO system, showing effectiveness within a range of ±10° for a roll and pitch angles and a full range on yaw angle and altitude. The PID controller module is designed to be used with commercial quadcopters and it has been implemented using inertial and ultrasonic sensors. Furthermore, the system also features a wireless interface to observe the aircraft performance during the flight.

Keywords: Quadcopter, control systems, PID controllers, UAV.

1. INTRODUCTION

The development and investigation of autonomous flight systems have increased lately because of the increasing amount of applications of unmanned aerial vehicle (UAV) in military fields such as intelligence, surveillance, and reconnaissance missions, and in civil fields like aerial surveillance, aerial photography and video, firefighting, and many others that are emerging (Natraj *et al.*, 2013; Nex & Remondino, 2014). In all cases, the incentive for the investigation in this field is the necessity to replace human intervention on high-risk jobs or missions.

In modern times, there is a wide variety of aircrafts used for UAVs, like fixed-wing airplanes, airships, and helicopters, among others (Benavidez *et al.*, 2014).Nevertheless, the platforms that have caught a lot of attention for investigation projects are multirotor helicopters, in particular quadrotors, because these aircrafts have a lot of advantages with respect to other aircrafts. Quadcopters have a simple structure and a great flight capacity and maneuverability, such as vertical take-off and landing and hovering flight (Escamilla, 2010).

A quadcopter has four rotors, each one of which has independent speed, allowing a balanced variation of the rotors' speed and thus generating the thrust and accelerations in the desired directions. Quadcopters are six-degrees-of-freedom (DoF) systems, what means that they can move along the three space axes X, Y and Z, and turn on the aircraft’s body axes describing roll (φ), pitch (θ) and yaw (ψ) angles. However, in the aircraft, there is only direct control over four DoF, corresponding to the altitude and the three angles: roll, pitch and yaw. Roll and pitch movements also generate motion along the Y and X axes of space, respectively.

Despite the advantages of quadcopters, like other aircrafts, they are extremely unstable systems, and so any imbalance in their motion (especially in roll and pitch) generates angular and linear accelerations, which can cause a collision if not compensated for quickly. Moreover, the quadcopter system is nonlinear and the movements are coupled with each other. PID controllers offer a simple but effective solution to stabilize the aircraft because they make it possible to treat every variable independently within a limited range in which the behavior of the quadcopter is approximately linear (Bouabdallah *et al.*, 2005; Castillo *et al.*, 2005).

This work presents the development of a quadcopter’s model considering its dynamic behavior, after which, based on an analysis of its involved variables a simplification is made, which makes easier the implementation of PID controllers on a SISO system.

2. MODELLING

PID controller design requires prior modeling of the system to know its behavior. Quadrotors have their four propellers placed on the ends of a cross-like structure. To maintain the balance of the overall torque, one pair of rotors spins in a clockwise direction while the remaining pair spins in a counter-clockwise direction. The speed of every rotor is controlled independently to generate the thrust and torque to move the aircraft. According to the orientation of the motion, there is an “x” mode and a “+” mode (see Fig. 1).

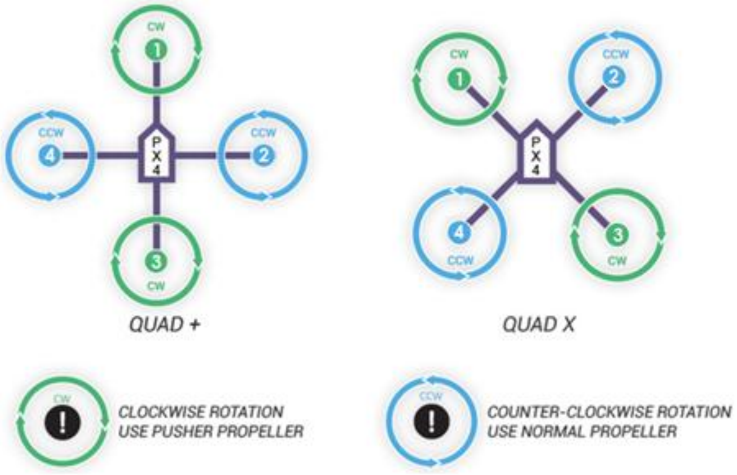
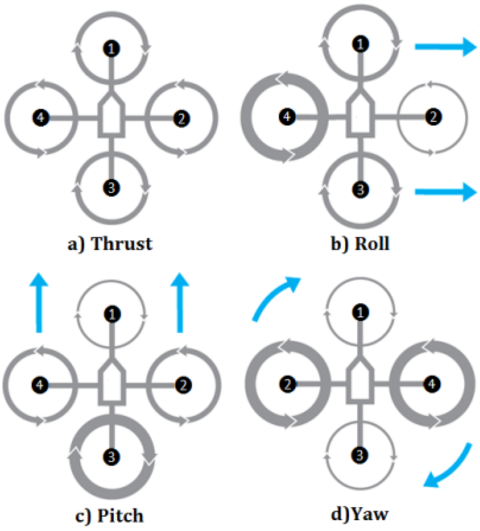


Figure 19. Modes "+" y "x" of the quadcopter.

The “+” mode is used in the present project, and the rotors in this mode are identified as ❶ Fontal, ❷Right, ❸Rear, and ❹Left. This way, the system’s inputs are defined as vertical thrust, roll, pitch and yaw ($\mathrm{U}_1, \mathrm{U}_2, \mathrm{U}_3, \mathrm{U}_4$ respectively). The vertical thrust (U_1) is produced when all four rotors increase or decrease their speed by the same amount. Roll (U_2) movement is produced when ❷ increases its speed and ❹decreases its speed (or vice versa), both by the same amount, so that the overall torque remains null and the thrust maintains its value. Pitch (U_3) movement is similar to roll but with the remaining rotor pair (❶and ❸. For the yaw (U_4) movement, the clockwise rotor pair (❶ and ❸ increases its speed while the remaining pair (❷and ❹ decreases its speed (or vice versa) by the same amount so that the overall torque is changed, producing a yaw movement, but the vertical thrust stays constant (De Lellis, 2011; Hald & Hesselbæk, 2005; Bresciani, 2008; Jiřinec, 2011).



a) Thrust

b) Roll

c) Pitch

d)Yaw

Figure 20. Quadcopter movements a) Thrust, b) Roll, c) Pitch, and d) Yaw.

For the quadcopter modeling, some reference systems need to be defined so that variables can be correctly identified. The EF system (Earth-Fixed Frame) is a system that is fixed in attitude and position, placed on earth with its X axis pointing North, Y axis pointing West, and Z axis perpendicular to both, pointing up. The BF system (Body-Fixed Frame) is a fixed system on the aircraft, its X axis pointing forward along the body, its Y axis in the left direction of the aircraft, and its Z axis perpendicular to both (upwards). Because of the motion of BF with respect to EF, the variables represented in EF are identified with an E subscript, and the variables in BF with a B subscript (Bresciani, 2008).

2.1. Kinematics

The linear kinematics of the quadcopter is summarized in (1) (Bresciani, 2008):

$$\dot{\Gamma}_E = R_\Theta V_B \tag{1}$$

where $\dot{\Gamma}_E$ is the vector of linear velocities with respect to EF, and V_B is the vector of linear velocities with respect to BF. R_Θ is the rotation matrix obtained from the product of the three basic rotation matrices. In equations (2), the expressions have been abbreviated as $s\theta = \sin\theta$, $c\theta = \cos\theta$, and $t\theta = \tan\theta$.

$$\boldsymbol{R}_\Theta = \begin{bmatrix} c\theta\, c\psi & s\varphi\, s\theta\, c\psi - c\varphi\, s\psi & c\varphi\, s\theta\, c\psi + s\varphi\, s\psi \\ c\theta\, s\psi & s\varphi\, s\theta\, s\psi + c\varphi\, c\psi & c\varphi\, s\theta\, s\psi - s\varphi\, c\psi \\ -s\theta & s\varphi\, c\theta & c\varphi\, c\theta \end{bmatrix} \tag{2}$$

The angular kinematics is summarized in (3):

$$\dot{\Theta}_E = T_\Theta \omega_B \tag{3}$$

where $\dot{\Theta}_E$ is the vector of angular velocities with respect to EF, and ω_B is the vector of angular velocities with respect to BF. T_Θ (equation 4) is the transfer matrix used to project angular velocities from EF to BF (Bresciani, 2008).

$$\boldsymbol{T_\Theta} = \begin{bmatrix} 1 & s\varphi\, t\theta & c\varphi\, t\theta \\ 0 & c\varphi & -s\varphi \\ 0 & \dfrac{s\varphi}{c\theta} & \dfrac{c\varphi}{c\theta} \end{bmatrix} \tag{4}$$

2.2. *Dynamics*

Describing the quadcopter dynamics is easier using a hybrid reference system which uses EF for the linear components of the model and BF for its angular components. The force contribution in the system generates linear movements, and so they are defined in EF. The general equation of forces according to Newton’s laws is summarized in (5):

$$\boldsymbol{F_E} = m\, \boldsymbol{\ddot{r}_E} \tag{5}$$

where, $\mathrm{F_E}$ is the force vector on EF, $\ddot{\Gamma}_\mathrm{E}$ is the vector of linear acceleration of the quadcopter on EF, and m is the mass of the quadcopter. To determine the force model, every force acting on the aircraft needs to be considered. The gravitational force on EF is a constant vector, because its direction its always pointing downward along the Z axis of EF. The gravity value is represented with g.

$$\boldsymbol{G_E} = \begin{bmatrix} 0 \\ 0 \\ -m\, g \end{bmatrix} \tag{6}$$

The thrust generated by the propellers is a direction constant vector on the Z axis of BF. To obtain the thrust on EF, the rotation matrix R_Θ must be used (Bresciani, 2008):

$$\boldsymbol{T_E} = \boldsymbol{R_\Theta T_B} = \boldsymbol{R_\Theta} \begin{bmatrix} 0 \\ 0 \\ T \end{bmatrix} = \begin{bmatrix} (s\psi\, s\varphi + c\psi\, s\theta\, c\varphi)\, T \\ (-c\psi\, s\varphi + s\psi\, s\theta\, c\varphi)\, T \\ (c\theta\, c\varphi)\, T \end{bmatrix} \tag{7}$$

where, $\mathrm{T_E}$ is the thrust on EF, $\mathrm{T_B}$ is the thrust on BF, and T is the scalar value of the thrust generated by the quadcopter. The value of T is given by (8), where b is the aerodynamic lift coefficient, and Ω_1 the speed of each rotor (Bresciani, 2008).

$$T = U_1 = b\left({\Omega_1}^2 + {\Omega_2}^2 + {\Omega_3}^2 + {\Omega_4}^2\right) \tag{8}$$

The force contribution on the quadcopter results in the system of equations in (9), which describes the linear dynamics of the aircraft on EF.

$$\begin{aligned} \ddot{x} &= (\sin\psi\ \sin\varphi + \cos\psi \sin\theta\ \cos\varphi)\frac{T}{m} \\ \ddot{y} &= (-\cos\psi \sin\varphi + \sin\psi \sin\theta \cos\varphi)\frac{T}{m} \\ \ddot{z} &= -g + (\cos\theta \cos\varphi)\frac{T}{m} \end{aligned} \tag{9}$$

The angular dynamics use BF and analyze the torque that generates angular acceleration on the system. The general equation for the torque on a body is given by (10): (Bresciani, 2008).

$$\tau_B = I\, \dot{\omega}_B + \omega_B \times (I\, \omega_B) \tag{10}$$

where, τ_B is the vector of torques acting on a body on BF, I is the body inertia, better known as the matrix of inertia, and $\dot{\omega}_\mathrm{B}$ is the vector of angular acceleration on BF. Because of the assumptions made that the origin of BF matches the aircraft's center of gravity, and the axes of the quadcopter also match the axes of BF, I is defined as:

$$\boldsymbol{I} = \begin{bmatrix} I_{XX} & 0 & 0 \\ 0 & I_{YY} & 0 \\ 0 & 0 & I_{ZZ} \end{bmatrix} \tag{11}$$

Equation (12) is obtained from (10), using (11).

$$\begin{aligned} \tau_x &= I_{XX}\dot{p} + q\,r\,(I_{ZZ} - I_{YY}) \\ \tau_y &= I_{YY}\dot{q} + p\,r\,(I_{XX} - I_{ZZ}) \\ \tau_z &= I_{ZZ}\dot{r} + p\,q\,(I_{YY} - I_{XX}) \end{aligned} \tag{12}$$

where, $\mathrm{p,q,r}$ are the angular velocities on BF.

The spinning rotors also produce a gyroscopic effect whose contribution to the torque model is given by (13): (Bresciani, 2008).

$$\boldsymbol{O_B}\,\Omega = J_{TP}\begin{bmatrix} -q \\ p \\ 0 \end{bmatrix}(-\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4) \tag{13}$$

where, $\mathrm{O_B}$ is the momentum vector produced by the gyroscopic effect on BF, $\mathrm{J_{TP}}$ is the total momentum of inertia on the rotor shaft, ω_B is the vector of angular velocities, and Ω is the total algebraic sum of the rotors' speeds (Bresciani, 2008).Torque generated by the propellers is summarized in the system of equations in (14), obtained by the operation principle of the quadcopter (Hald & Hesselbæk, 2005; Bresciani, 2008; Jiřinec, 2011).

$$\begin{aligned} \tau_\varphi &= U_2 = b\,l\left({\Omega_4}^2 - {\Omega_2}^2\right) \\ \tau_\theta &= U_3 = b\,l\left({\Omega_3}^2 - {\Omega_1}^2\right) \\ \tau_\psi &= U_4 = d\left({\Omega_2}^2 + {\Omega_4}^2 - {\Omega_1}^2 - {\Omega_3}^2\right) \end{aligned} \tag{14}$$

As shown in (14), the torques produces by the propellers are directly associated with the system inputs. The torque contribution gives the angular dynamics of the system (equations 15).

$$\begin{aligned} \dot{p} &= \frac{(I_{YY} - I_{ZZ})}{I_{XX}}\,q\,r + \frac{J_{TP}}{I_{XX}}\,q\,\Omega + \frac{\tau_\varphi}{I_{XX}} \\ \dot{q} &= \frac{(I_{ZZ} - I_{XX})}{I_{YY}}\,p\,r + \frac{J_{TP}}{I_{YY}}\,p\,\Omega + \frac{\tau_\theta}{I_{YY}} \\ \dot{r} &= \frac{(I_{XX} - I_{YY})}{I_{ZZ}}\,p\,q + \frac{\tau_\psi}{I_{ZZ}} \end{aligned} \tag{15}$$

The total quadcopter dynamics model results in the system of equations in (16) (Bresciani, 2008).

$$\begin{aligned} \ddot{x} &= (\sin\psi\ \sin\varphi + \cos\psi \sin\theta\ \cos\varphi)\frac{T}{m} \\ \ddot{y} &= (-\cos\psi \sin\varphi + \sin\psi \sin\theta \cos\varphi)\frac{T}{m} \\ \ddot{z} &= -g + (\cos\theta\cos\varphi)\frac{T}{m} \\ \dot{p} &= \frac{(I_{YY} - I_{ZZ})}{I_{XX}}\,q\,r + \frac{J_{TP}}{I_{XX}}\,q\,\Omega + \frac{\tau_\varphi}{I_{XX}} \\ \dot{q} &= \frac{(I_{ZZ} - I_{XX})}{I_{YY}}\,p\,r + \frac{J_{TP}}{I_{YY}}\,p\,\Omega + \frac{\tau_\theta}{I_{YY}} \end{aligned} \tag{16}$$

$$\dot{r} = \frac{(I_{XX} - I_{YY})}{I_{ZZ}}\, p\, q + \frac{\tau_{\psi}}{I_{ZZ}}$$

In the model shown above, the dynamic model of the DC motors has been disregarded because it is much faster than the quadcopter dynamics.PID controllers are only suitable for linear systems, so the model must be linearized around an operating point, which is hover flight. Using small-angle approximations and disregarding the equations of the X and Y movements that are not used in the control, the system of equations of (17) is obtained (Bresciani, 2008).

$$\begin{aligned} \ddot{z} &= -g + \frac{T}{m} \\ \ddot{\varphi} &= \frac{\tau_{\varphi}}{I_{XX}} \\ \ddot{\theta} &= \frac{\tau_{\theta}}{I_{YY}} \\ \ddot{\psi} &= \frac{\tau_{\psi}}{I_{ZZ}} \end{aligned} \tag{17}$$

The original model and the linear model have similar behavior for small-angle inputs because the coupling between variables becomes insignificant. The system shows practically double integrator behavior, the response of which is a parabolic tendency even for small step inputs.

3. PID CONTROLLER DESIGN

3.1. Architecture

For PID controller design, parallel architecture has been used (Åström & Hägglund, 2009; see Fig. 3).

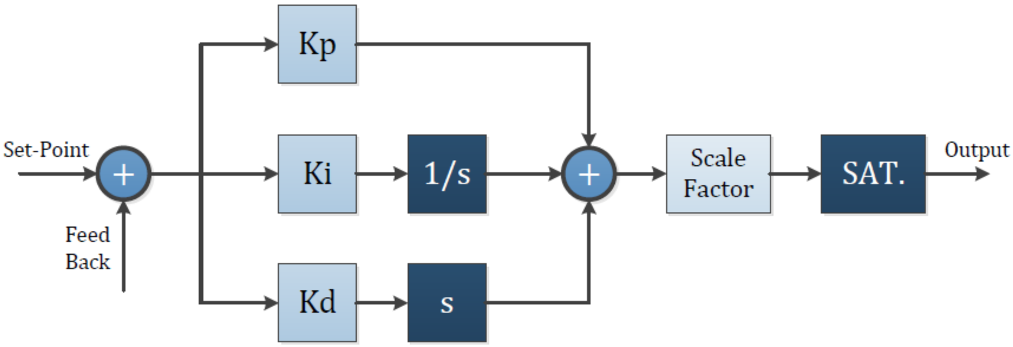


Figure 21. PID parallel architecture.

In the diagram, there is an additional block corresponding to a scale factor, which compensates the aerodynamic parameters of the quadcopter so that the PID design becomes simpler. This block also scales the control signal to values compatible with the existing hardware. Finally, at the output there is a signal saturation block. Roll, pitch, yaw, and altitude controllers have a similar architecture with minimal changes, especially for yaw and altitude.

The yaw controller reads the commands received from the RC transmitter as angular velocities to improve the movement of the system and avoid oversensitivity with yaw movement. Thus, on the controller input there is an integrator which translates the angular velocity into an angular set-point. Additionally, in yaw control, the discontinuities of the system in 360° and -360° must be taken into account and corrected to avoid misbehavior of the system.

For altitude control, the architecture is almost the same, with the difference that on the output of the PID there is a summation device used to compensate for the value of gravity.

3.2. Design

In the Laplace domain, the equations of the linear model can be transformed as shown below for the roll angle (equation 18).

$$\ddot{\varphi} = \frac{\tau_\varphi}{I_{XX}} \rightarrow \varphi(s) = \frac{1}{s^2}\frac{\tau_\varphi}{I_{XX}} \tag{18}$$

This way, each equation of the model can be treated as a double integrator, and at the output of the controller, the aerodynamic coefficients and the inertia affecting the transfer function can be compensated.

Once the system is considered as a double integrator, the controller design is simplified and the same design can theoretically be used for all variables. The PID design is easily done using SISOTOOL of MATLAB, which let the user place poles and zeroes, as well as to adjust the controller gain until the response of the closed-loop system is acceptable. Using the MATLAB tool, the compensators had the transfer function shown in (19), corresponding to a PD controller.

$$G_{PD}(s) = 0{,}8\ (1 + 10\ s) \tag{19}$$

Given that the system is a double integrator, theoretically, the steady-state error is null, which is the why the controller is only PD.

For the controller implementation, the continuous function must be discretized so that it can be applied in a digital microcontroller. Despite the controller design using the math model, the actual gains can change in practice, so it is better to implement the general equation of a discrete PID controller (20) (Basdogan, 2004).

$$u[k] = u[k-1] + \left(K_p + \frac{K_d}{T_s}\right) e[k] + \left(-K_p - 2\,\frac{K_d}{T_s}\right) e[k-1] + \left(\frac{K_d}{T_s}\right) e[k-2] \tag{20}$$

4. IMPLEMENTATION

The measurement of roll, pitch and yaw angles is made using a low-cost AHRS (Attitude and Heading Reference System), which is integrated by accelerometers, gyroscopes, and magnetometers along with a built-in Extended Kalman Filter to estimate the angles. The AHRS used is the CHR-UM6, which gives the angle information through a serial interface using a data package with a specific and simple-to-decode frame.

Altitude measurement is generally done with pressure sensors. However, at great heights there are frequent sudden pressure variations that lead to mistaken measurements. For this reason, a PING ultrasonic sensor from Parallax, which measures distances using ultrasonic waves, is used in the project. The drawback of the sensor, however, is that it has a limited measurement range to a maximum of 3 meters, and so the altitude controller is also limited to that height. The ultrasonic sensor gives a pulse train signal, in which the pulse width corresponds to the travel time of the ultrasonic waves, and therefore it can be scaled to find the distance.

Data transmission between the quadcopter and the land station (PC) is made through a wireless link using XBee communication modules. These modules communicate with each other with a ZigBee protocol (IEEE 802.15.4) over the free 2.4GHz frequency. The interface between XBee modules and other devices is serial, so it is simple to integrate them on any system.

The program in the microcontroller consists of three blocks: the main block, and two interruption blocks triggered by a serial reception of the AHRS and the land station (PC). During the AHRS serial interruption, the package is read, and Euler angles, accelerations, and angular velocities data are stored for further processing and filtering in the main program. In the serial interruption triggered by the PC, the settings and calibration data is read to be processed later. The main block of the program is also composed by several algorithm blocks as shown in Fig. 4.

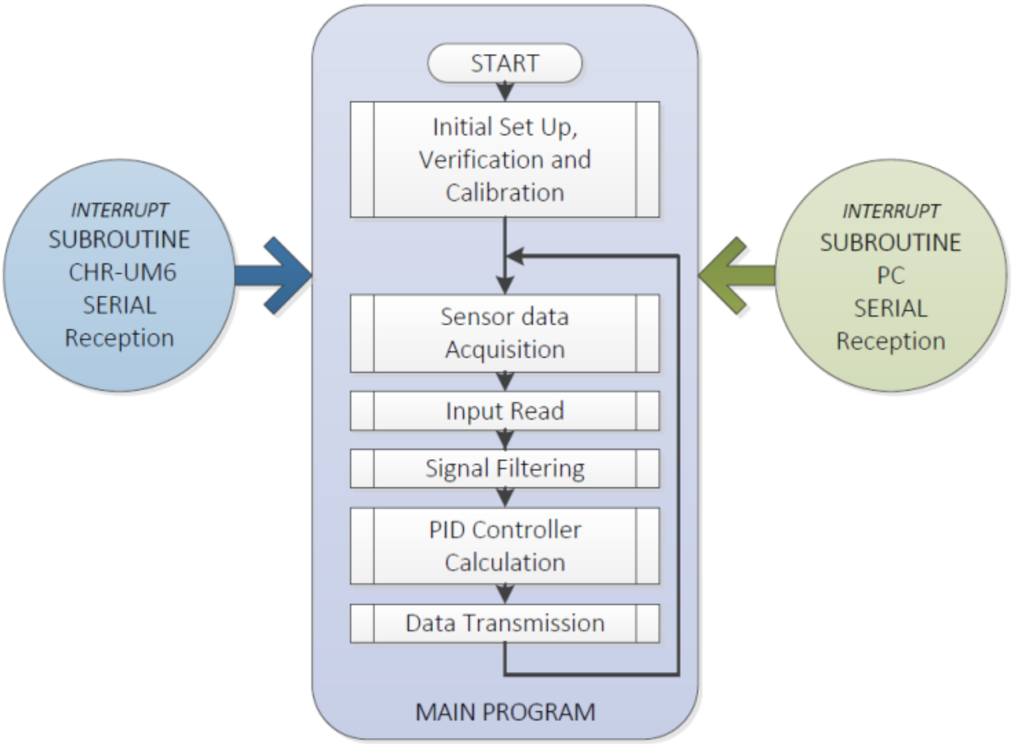

START
Initial Set Up, Verification and Calibration
Sensor data Acquisition
Input Read
Signal Filtering
PID Controller Calculation
Data Transmission
MAIN PROGRAM
INTERRUPT SUBROUTINE CHR-UM6 SERIAL Reception
INTERRUPT SUBROUTINE PC SERIAL Reception

Figure 4. Control System flowchart.

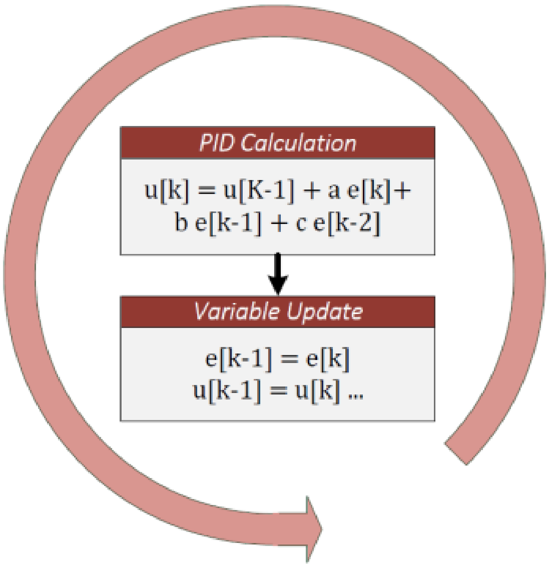


Figure 5. PID calculation chart.

During the system set up, some subroutines are executed to verify the status of sensors and to correct errors to avoid further problems in flight.

Once the system is started, the microcontroller is waiting to receive data from the AHRS or the PC. The AHRS is set to transmit angle data with a fixed frequency of 50Hz, so when data is received, the control loop is executed at the same frequency to synchronize operations. The control loop first processes the data received from the AHRS, than measures the altitude and the inputs from the RC receiver are read. Afterwards, all signals are filtered to remove the noise produced mostly by the aircraft's vibration. After the signal filtration, the PID outputs are computed, and finally, the data transmission subroutine is executed. PID controllers and filter algorithms are implemented using difference equations. In every control loop, the filters and PID controllers are computed, and afterwards the variables are updated, as shown in Fig. 5.

The serial link between the PC and the quadcopter has been mainly set to change the settings of the controllers and adjust their gains. When a land station package is available, the system processes it only when the aircraft has landed, because sudden setting changes may produce misbehaviors of the system during flight.

For the supervision of the aircraft and controllers' performance, a graphical interface or HMI has been implemented using LabVIEW (see Fig. 6), which simulates a virtual cockpit with instruments similar to a real aircraft that show the current state of the quadcopter (see Fig. 7).

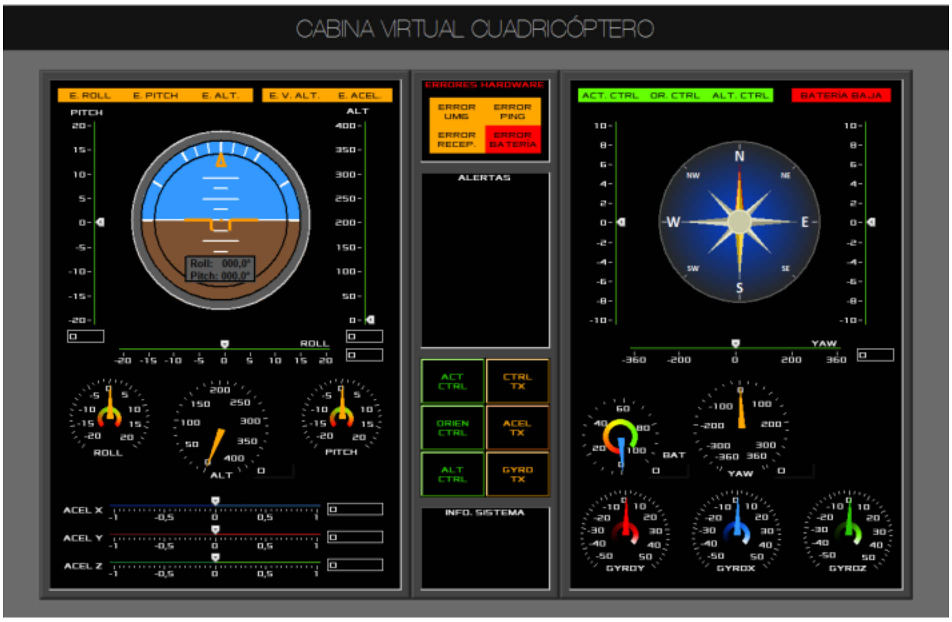


Figure 6. Virtual cockpit on the HMI.

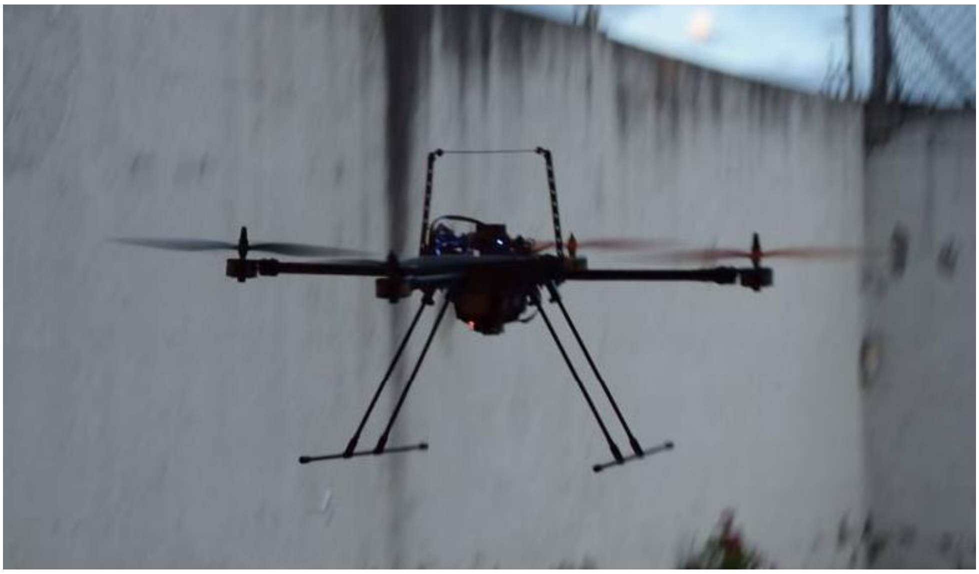


Figure 7. Quadcopter.

5. TESTS AND RESULTS

In the tests of the controllers during flight, the gains were adjusted, because the initial values of the design did not show a suitable performance. The derivate gains in particular had to be drastically decreased because their effect destabilized the system. A mild permanent oscillation of the aircraft and the large derivative effect produced a bigger oscillation which led to the system destabilization.

For roll, the proportional gain was set to Kp = 0.75, while the proportional gain for pitch resulted in Kp = 0.7. In both cases, the derivative gain was reduced to Kd = 0.02 and the integral gain was increased to Ki = 0.4 due to an error in steady state.

The proportional gain of the yaw controller was increased to Kp = 0.9; the derivative gain was decreased to Kd = 0.05 and the integral gain was set to Ki = 0.3. In yaw control it is important to consider the discontinuities of the signal and to correct the controller performance on those points to avoid system failures. The controller implemented corrects the discontinuities and its response is acceptable, as shown in Fig. 11.

The altitude controller's proportional gain was adjusted to Kp = 0.6; the derivative gain to Kd = 0.45; and the integral gain to Ki = 0.2. In altitude control, the ultrasonic sensor showed some mistakes which produced random signal peaks that become more frequent when the aircraft is higher, which is why a simple algorithm was implemented to remove the peaks in the signal.

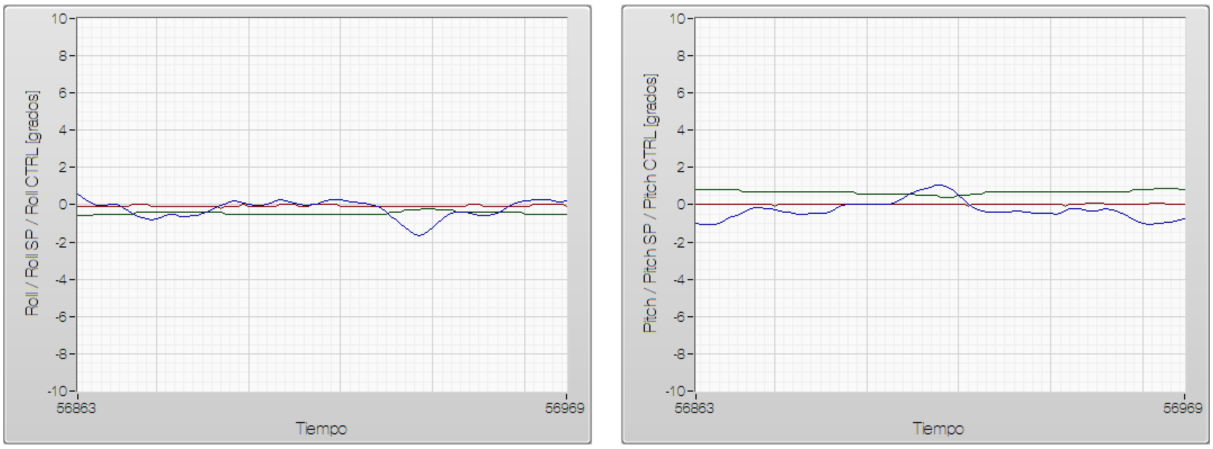


Figure 8. Roll and pitch angles in hover flight.

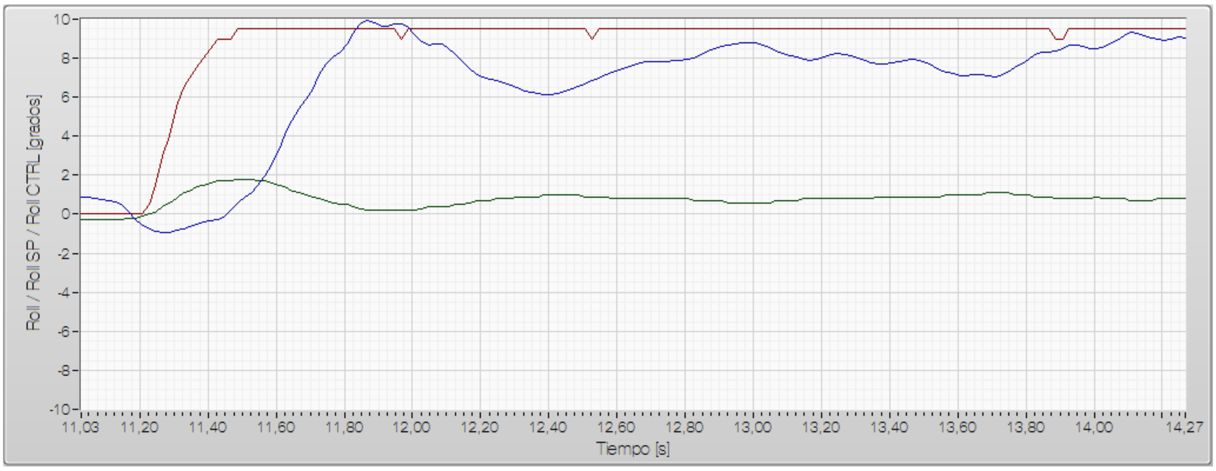


Figure 9. Roll step response.

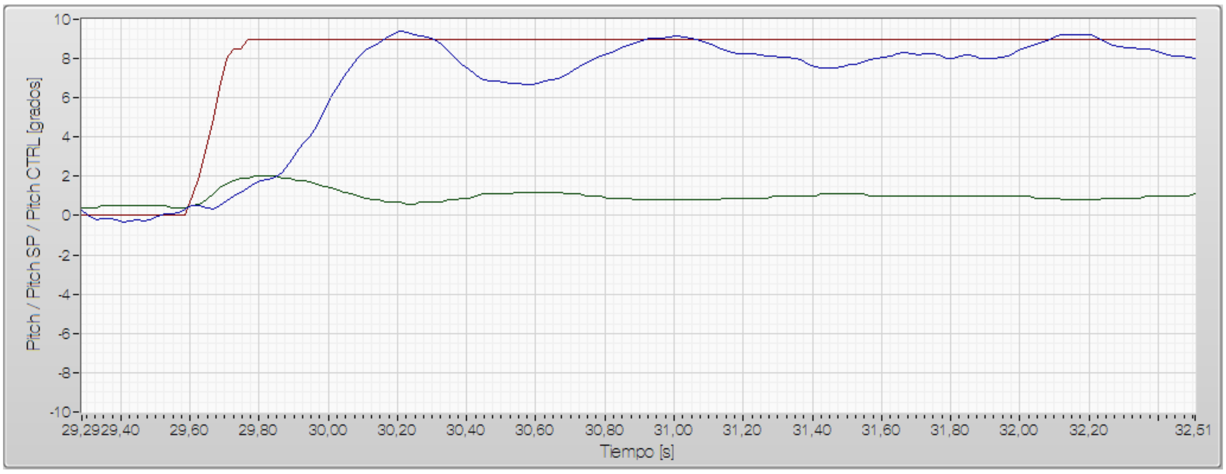


Figure 10. Pitch step response.

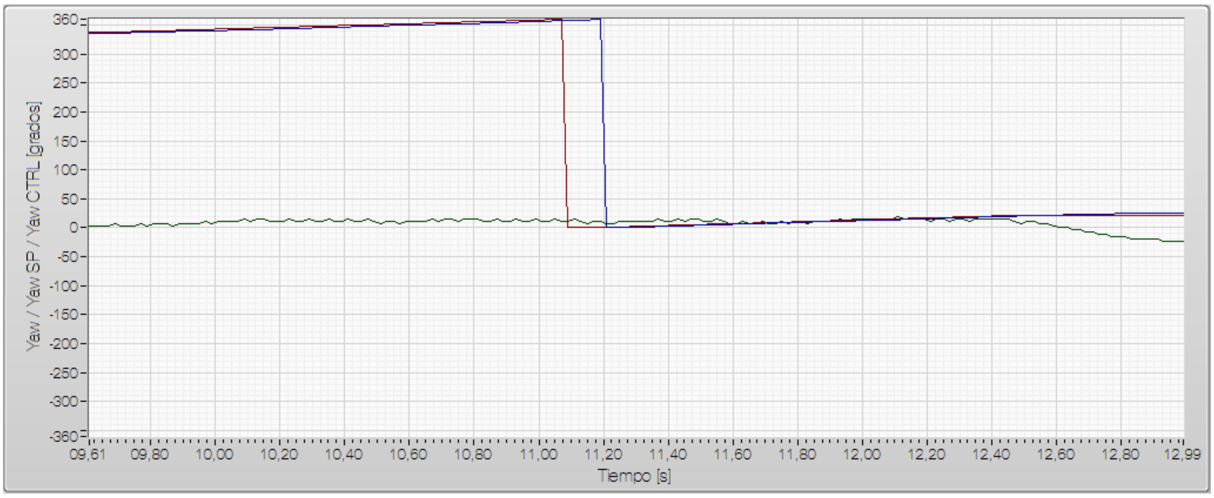


Figure 11. Yaw response in a discontinuity point.

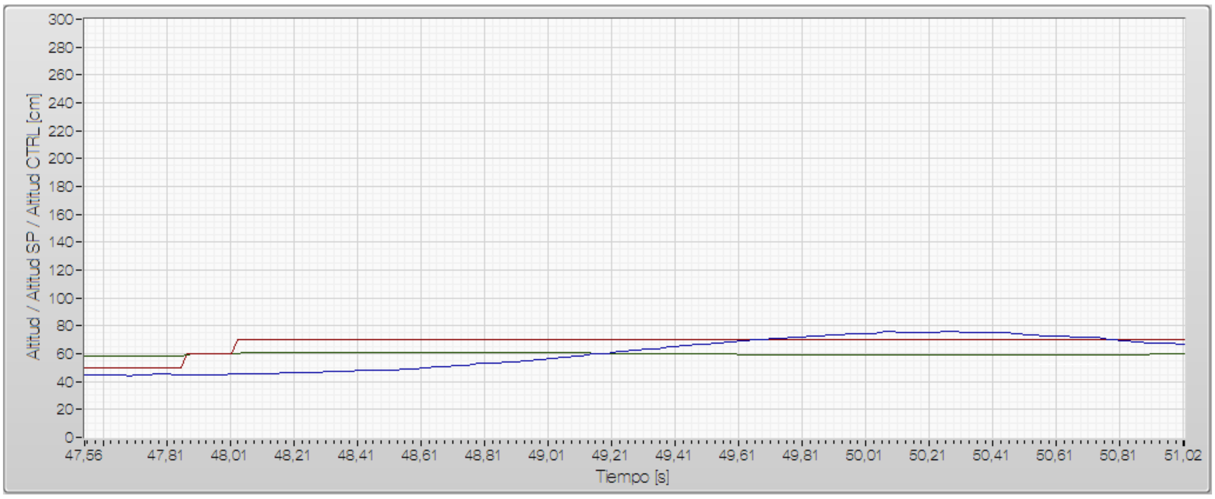


Figure 12. Altitude step response.

6. CONCLUSIONS

PID controllers are capable of stabilizing a complex system such as the quadcopter; however, the influence of intense and long external disturbances affects the behavior of the controller.

The actual gains of the PID controllers differ from the original design gains because the math model does not consider with exactitude all the effects acting on the quadcopter like the vibrations or the oscillations of the system. Despite the nature of the system, whose behavior is practically a double integrator, it has a steady state error, which is why the integral gains of all four controllers had to be increased to obtain an improved response. Derivative gains were strongly reduced due to the mild oscillation of the aircraft because its effect produces unstable control signals, which also produce unstable behavior in the system.

A land system with a graphical interface is helpful to supervise the performance of the quadcopter and the controllers during flight, and to adjust the controllers' gains or setting if needed.

To improve the system behavior, in future works, the implementation of non-linear controls can be made, which would be based on a non-simplified model, allowing wider ranges on roll and pitch angles.

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